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# DEPARTMENTS.

## SOLUTIONS OF PROBLEMS.

### ALGEBRA.

**265. Proposed by G. W. GREENWOOD, M. A., Dunbar, Pa.**

Obtain the reduced cubic $4\theta^3-I\theta+J=0$ of the biquadratic $ax^4+4bx^3+6cx^2+4dx+e=0$.

**I. Solution by the PROPOSER.**

Assume $ax^4+4bx^3+6cx^2+4dx+e\equiv a(x^2+2mx+p)(x^2+2nx+q)$.
$\therefore a(m+n)=2b$; $a(4mn+p+q)=6c$; $a(mq+np)=2d$; $apq=e$.
Let $amn=c-\theta$; then $a(p+q)=2c+4\theta$.
Substituting in the identity

$$\begin{vmatrix} 1 & 1 & 0 \\ m & n & 0 \\ p & q & 0 \end{vmatrix} \times \begin{vmatrix} 1 & 1 & 0 \\ n & m & 0 \\ q & p & 0 \end{vmatrix} \equiv \begin{vmatrix} 2 & m+n & p+q \\ m+n & 2mn & mq+np \\ p+q & mq+np & 2pq \end{vmatrix} \equiv 0$$

we obtain $\dfrac{8}{a^3}\begin{vmatrix} a & b & c+2\theta \\ b & c-\theta & d \\ c+2\theta & d & e \end{vmatrix}=0$; *i. e.*, $4\theta^3-I\theta+J=0$.

**II. Solution by L. E. NEWCOMB, Los Gatos, Cal.**

$$ax^4+4bx^3+6cx^2+4dx+e\equiv x^4+\frac{4bx^3}{a}+\frac{6cx^2}{a}+\frac{4dx}{a}+\frac{e}{a}=0\ldots(1).$$

Let $x=y/a$; then (1) becomes $y^4+4by^3+6acy^2+4a^2dy+a^3e=0\ldots(2)$.

$y^4+4by^3+6acy^2+4a^2dy+a^3e+(ay+\beta)^2=(y^2+2by+\lambda)^2$, by Ferrari's solution; whence, by equating coefficients of like powers of $y$, and eliminating $a$ and $\beta$, $\lambda^3-3ac\lambda^2+a^4(4bd-ae)\lambda-a^3(2d^2+2b^2e-3ace)=0$. Let $4^{\frac{1}{3}}\theta+ae=\lambda$, then
$4\theta^3-4^{\frac{1}{3}}a^2(3c^2+ac-4bd)\theta+a^3[c(4bd-ac-2c^2)-(2cd^2+2b^2e-3ace)]=0\ldots(3)$,

which is of the required form.

**266. Proposed by L. E. NEWCOMB, Los Gatos, Calif.**

Find the $n$th term and the sum of $n$ terms of the series $1+3+7+17+\ldots$

**I. Solution by THEODORE L. DeLAND, Treasury Department, Washington, D. C.**

Let $u_1$, $u_2$, $u_3$, $u_4$, ..., $u_n$ be the terms of the series, in the Calculus of Finite Differences, and let $\Delta u_1$, $\Delta^2 u_1$, $\Delta^3 u_1$, and $\Delta^4 u_1$ be the symbols for the different orders of differences.